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USING DIFFERENT NULL HYPOTHESES TO TEST FOR SEASONAL UNIT ROOTS IN ECONOMIC TIME SERIES

Antônio Aguirre
Andreu Sansó

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UNIVERSIDADE FEDERAL DE MINAS GERAIS
FACULDADE DE CIÊNCIAS ECONÔMICAS
CENTRO DE DESENVOLVIMENTO E PLANEJAMENTO REGIONAL

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Antônio Aguirre
'Departamento de Ciências Econômicas, Universidade Federal de Minas Gerais'
and researcher of CEDEPLAR/UFMG.

Andreu Sansó
'Departamento d'Econometria, Estadística i Economia Espanyola, Universitat de Barcelona'
and researcher of the 'Equip d'Anàlisi Quantitativa Regional' group.

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CONTENTS

I. INTRODUCTION

The study of the seasonal properties of economic time series has been the subject of considerable research in the last years [see, *inter alia*, Beaulieu and Miron (1992, 1993), Ghysels (1994), Harvey and Scott (1994), Hylleberg (1992), Hylleberg *et al*. (1993), Miron (1994), Osborn (1990)]. Concerning the Brazilian literature Aguirre (1997) analyzed a quarterly series of prices testing for the presence of seasonal unit roots in the data.[1] The evidence provided by these studies is that, in addition to being nonstationary at the zero frequency, many seasonally observed economic time series also display seasonal variations which are larger and far more irregular than previously thought of. As a matter of fact, while some variables show a deterministic seasonal pattern, others display seasonal movements that tend to change slowly over time (a mixed pattern also appears to be relatively common). In other words, seasonally recorded economic time series often appear to display *nonstationary stochastic seasonal variations* and, in those cases, the corresponding DGPs are referred to as *seasonally integrated* [2] or *seasonal unit root processes*.

When working with seasonally observed data sets, applied researchers may use some filter to obtain seasonally adjusted data (such an approach was followed by Haache (1974) in studying the demand for money) or may attempt to capture seasonality by means of seasonal dummies, which is equivalent to assume seasonal variations to be purely deterministic. "However, if seasonal effects change gradually over time, this (second) approach leads to dynamic misspecification ..." (Harvey and Scott, 1994, page 1324). For this reason, whenever seasonal data is used in econometrics, it seems advisable to test for the time series properties of the variables, since it is better to test rather than assume the appropriateness of any model specification.

By now the number of testing procedures designed to help distinguish between stationary and nonstationary stochastic movements (possibly around deterministic components) in seasonally observed economic time series is quite large. This transforms the choice of a testing strategy in a delicate task where the knowledge of pros and cons of each test is of paramount importance. The objective of this paper is to attempt to make a contribution by discussing the application of different testing procedures and techniques used in determining the seasonal properties of quarterly data. To attain this objective we test the same series analyzed by Aguirre (1997) but following another testing strategy and also show how to apply a different test which specifies a null hypothesis of stationarity - instead of the usual integration null -, a change that allegedly increases the power of the test.[3]

The paper is organized as follows. Section II sets out the usual broad classes of seasonal time series processes and three definitions of the concept of seasonal integration —indicating which one will be used in this paper. Section III provides a brief synopsis of some of the existing seasonal unit root tests. We focus, in particular, on the Hylleberg *et al*. (1990) [henceforth, HEGY] testing procedure because it is the most widely reported test for seasonal unit roots in the applied literature, and on the test statistics proposed by Canova and Hansen (1995) [hereafter, CH] who change the usual null hypothesis from nonstationarity to stationarity. The data description and test results are presented in Section IV. Section V concludes.

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[1] Oliveira and Picchetti (1997) also study a problem of seasonal integration and cointegration using Japanese data.

[2] A concept which may mean different things for different authors (see next Section).

[3] As will be discussed in a later section this allegation does not merit full credit.

II. SEASONAL PROCESSES AND SEASONAL INTEGRATION

The theory underlying seasonal time series analysis usually considers three broad classes of processes**:** purely deterministic seasonal processes, (covariance) stationary processes and integrated seasonal processes. The first class includes those processes generated by purely deterministic components such as a constant term, seasonal dummy variables and deterministic trends. In the following example the variable y_t - observed s times each year - is generated solely by seasonal intercept dummies**:**

$$y_t = \sum_{i=1}^{s} \alpha_i Di_t + \varepsilon_t \qquad (1)$$

where the Di_t (i = 1, 2,..., s) take value 1 when t lies on season i, and zero otherwise, and ε_t is a series of IID random variables. This equation can be reformulated so as to avoid confounding the levels and the seasonals, in the following way:

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i^* Di_t^* + \varepsilon_t \qquad (2)$$

where μ is the mean of the process and the coefficients α_i^* are constrained to sum zero. In order to make this constrain operative the Di_t^* dummies are defined to be 1 when t lies in season i, –1 when t lies in season s and zero otherwise. Finally, the above equation may also include deterministic trends with constant or variable coefficients across seasons, *i.e.*

$$y_t = \mu + \sum_{i=1}^{s-1} \alpha_i Di_t + \sum_{i=1}^{s} \beta_i [Di_t \times g(t)] + \varepsilon_t \qquad (3)$$

where $g(t)$ is a deterministic polynomial in t.[4]

The second case - covariance stationary seasonal process - can be exemplified by the model expressed as

$$y_t = \rho_s y_{t-s} + \varepsilon_t \qquad (4)$$

where $|\rho_s| < 1$ and ε_t is a series of IID random variables.

If $\rho_s = 1$ in equation (4), we have a *seasonal random walk*, a process that exhibits a seasonal pattern which varies over time. This is the third class of seasonal process listed above. In that case, $\Delta_s y_t$, defined as

$$\Delta_s y_t = y_t - y_{t-s} \qquad (5)$$

[4] Note that all the above deterministic processes will never change their shape and can be forecast.

is stationary. The main difference between these forms of seasonality is that in the deterministic model, if ε_t is white noise, shocks have only an immediate impact; in the stationary seasonal model shocks have a transitory effect (they die out in the long run), while they have a permanent effect in the integrated model. That is to say, seasonally integrated processes have properties similar to those observed in the ordinary (zero frequency) integrated series. "...they have 'long memory' so that shocks last forever and may in fact change permanently the seasonal patterns. They have variances which increase linearly since the start of the series and are asymptotically uncorrelated with processes with other frequency unit roots" (Hylleberg *et al.*, 1990, p. 218).

Ilmakunnas shows that "the testing sequence depends on the definition of seasonal integration adopted" (Ilmakunnas, 1990, page 97). Out of the different existing definitions of seasonal integration we mention three of them**:** one proposed by Osborn *et al*. (1988), another one due to Engle *et al*. (1989) and a final one given by Hylleberg *et al.* (1990). Ilmakunnas reproduces the first two definitions**:** according to the first one a variable is said to be integrated of orders (d,D) —denoted I(d,D)— if the series becomes stationary after first differencing *d* times and seasonal differencing *D* times, that is to say, X_t ~ I(d,D) if $(1-L)^d(1-L^s)^D X_t = \Delta^d \Delta_s^D X_t$ is stationary.[5] The second concept states that a time series is integrated of order d_0 and d_s, denoted SI(d_0,d_s), if $(1-L)^{d_0}[S(L)]^{d_s} X_t = \Delta^{d_0}[S(L)]^{d_s} X_t$ is stationary, where the polynomial expression *S(L)* is defined as $S(L) = 1 + L + L^2 + \ldots + L^{S-1}$.

When variables do not present seasonal integration both definitions coincide, *i.e*., I(1,0) = SI(1,0), I(2,0) = SI(2,0), etc. On the contrary, whenever a series is seasonally integrated these definitions differ. This is so because $\Delta_s = (1-L^s)$ can be factored into $(1-L)S(L)$. In this way, the equivalent of I(0,1) is SI(1,1); I(1,1) = SI(2,1), and so on. In the same way, the SI(0,1) process —using Engle's definition— does not have an equivalent one if we use Osborn's concept. The SI definition will be used in this paper.

Finally, a third definition asserts that "a series x_t is an integrated seasonal process if it has a seasonal unit root in its autoregressive representation. More generally it is integrated of order *d* at frequency θ if the spectrum of x_t takes the form

$$f(\omega) = c(\omega - \theta)^{-2d}$$

for ω near θ. This is conveniently denoted by $x_t \sim I_\theta(d)$" (Hylleberg *et al.* 1990, p. 217). This definition is convenient when discussing the results of some tests, as will be shown in Section IV.

[5] *L* is the usual lag operator.

III. SEASONAL UNIT ROOT TESTS

Before describing the different tests that will be used in this paper we will mention the testing strategy proposed by Dickey and Pantula (1987)**:** these authors suggest that, in order to preserve the nominal test size, in the case of more than one (zero frequency) unit root, it is convenient to start the testing sequence from the maximum number of roots under consideration.[6] Ilmakunnas (1990) conjectures that this also holds when working with quarterly data[7] and presents a whole sequence of possible tests that starts with the SI(2,1) case, indicating which alternative can be tested in each case.

If a variable has to be filtered in some way to make it stationary, this may be caused by a zero frequency unit root (corresponding to $(1 - L)$), or by seasonal frequency unit roots [corresponding to the decomposition of *S(L)*]. This fact determines the regression model to be estimated in order to test a given null hypothesis. "The basic idea is that when the maintained hypothesis is that there is a unit root at lag 1 or at seasonal lag, the available test statistics are modified so that appropriately differenced (Δ or Δ_4, respectively) data is used when running the test regression. When the maintained hypothesis is seasonal frequency unit roots, seasonally averaged [*S(L)*-form] data is used" (Ilmakunnas, 1990, page 80).

III.1. Tests with null of nonstationarity

As our series is quarterly, it is possible to *postulate that there is no seasonal integration* and use an ADF test (Dickey and Fuller, 1979, 1981) to check for the presence of zero frequency unit roots. Regressions [1] and [2] of Table I can be fit to the data in order to test the null indicated in column 3 against the alternative written in column 4. If properly filtered series enter the relevant test regressions, it is possible to test for the presence of zero frequency unit roots using the same test but *postulating the existence of seasonal unit roots* (see models [3] and [4] in Table I). In these last two cases, the (postulated) seasonal unit roots appear in the null as well as in the alternative hypothesis since the ADF test can only check for the presence (and number) of zero frequency unit roots.

A test for seasonal integration which resembles a generalization of the ADF test was proposed by Dickey, Hasza and Fuller (1984) (from now on referred to as DHF), by setting a test of the hypothesis $\rho_s = 1$ against the alternative $\rho_s < 1$ in the model $y_t = \rho_s y_{t-s} + \varepsilon_t$. This DHF test —as well as similar ones proposed in the following years— only allows for unit roots at all of the seasonal frequencies and has an alternative hypothesis which imposes a strong restriction on the roots. "A major drawback of this (DHF) test is that it doesn't allow for unit roots at some but not all of the seasonal frequencies and that the alternative has a very particular form, namely that all the roots have the same modulus" (Hylleberg *et al.*, 1990, page 221).

Trying to overcome the above mentioned drawbacks HEGY propose a more general test strategy that allows for unit roots at some (or even all) of the seasonal frequencies as well as the zero frequency. In order to test the hypothesis that the roots of the autoregressive polynomial lie on the unit circle against the alternative that they lie outside of it, these authors use - in the case of quarterly data - the following factoring of the polynomial:

[6] This is a different procedure from that recommended by Charemza and Deadman (1992, p. 137) and followed by Aguirre (1997).

[7] This conjecture was proved to be true not only for quarterly but also for monthly data by Franses and Taylor (1997).

TABLE I

Unit Root Tests for Different Hypotheses

Eq. #	Description of the tests	Null hypotheses	Alter native Hypo theses	Remarks
[1]	ADF for ΔX_t : $\Delta^2 X_t = \mu_t + \beta \Delta X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta^2 X_{t-j} + \varepsilon_t$ (*)	SI(2,0)	SI(1,0)	
[2]	ADF for X_t : $\Delta X_t = \mu_t + \beta X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta X_{t-j} + \varepsilon_t$ (**)	SI(1,0)	SI(0,0)	
[3]	ADF for $\Delta_4 X_t$: $\Delta\Delta_4 X_t = \mu_t + \beta\, \Delta_4 X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + \varepsilon_t$ (*)	SI(2,1)	SI(1,1)	
[4]	ADF for $S(L)X_{t-1}$: $\Delta_4 X_t = \mu_t + \beta\, S(L) X_{t-1} + \sum_{j=1}^{p} \alpha_j \Delta_4 X_{t-j} + \varepsilon_t$ (**)	SI(1,1)	SI(0,1)	
[5]	DHF for ΔX_t : $\Delta\Delta_4 X_t = \mu_t + \Delta X_{t-4} + \sum_{j=1}^{p} \alpha_j \Delta\Delta_4 X_{t-j} + \varepsilon_t$ (*)	SI(2,1)	SI(1,0)	
[6]	DHF for X_t : $\Delta_4 X_t = \mu_t + \beta X_{t-4} + \sum_{j=1}^{p} \alpha_j \Delta_4 X_{t-j} + \varepsilon_t$ (**)	SI(1,1)	SI(0,0)	
[7]	HEGY for ΔX_t : $\Delta\Delta_4 y_t = \mu_t + \pi_1 Z1_{t-1} + \pi_2 Z2_{t-1} + \pi_4 Z3_{t-1} + \pi_3 Z3_{t-2} + \sum_{i=1}^{k} \delta_i\, \Delta\Delta_4 y_{t-i} + \varepsilon_t$ (*)	SI(2,1) SI(2,1)	SI(1,1) SI(1,0)	π_1 tested; $\pi_2 = \pi_3 = \pi_4 = 0$ π_1, π_2, π_3, π_4 tested
[8]	HEGY for X_t : $\Delta_4 y_t = \mu_t + \pi_1 Y1_{t-1} + \pi_2 Y2_{t-1} + \pi_4 Y3_{t-1} + \pi_3 Y3_{t-2} + \sum_{i=1}^{k} \delta_i\, \Delta_4 y_{t-i} + \varepsilon_t$ (**)	SI(1,0) SI(1,1) SI(1,1)	SI(0,0) SI(1,0) SI(0,1)	π_1 tested; π_2, π_3, $\pi_4 \neq 0$ π_2, π_3, π_4 tested; $\pi_1 = 0$ π_1 tested; $\pi_2 = \pi_3 = \pi_4 = 0$

(*) μ_t may be zero, a constant, a set of dummies, or any combination of them. (**) μ_t may also include a linear trend.

$$\begin{aligned}(1-L^4) &= (1-L)(1+L)(1-iL)(1+iL)\\ &= (1-L)(1+L)(1-L^2)\\ &= (1-L)(1+L+L^2+L^3) \qquad (6)\\ &= (1-L)S(L)\end{aligned}$$

and, after making use of some results from algebra, they obtain an equivalent expression which facilitates the testing of hypotheses. The resulting testable model that can be used to check for the presence of two unit roots at the zero frequency and seasonal unit roots at seasonal frequencies is given by equation [7] of Table I, which can be estimated by OLS and the statistics on the π's used for inference, and where:

μ_t may contain a constant, a deterministic trend and/or three seasonal dummies;

$Z1_t$ = $\Delta_4 X_t = \Delta[S(L)X_t] = S(L)[\Delta X_t]$ is the transformation of ΔX_t retaining the unit root at the zero frequency;

$Z2_t = [-(1 - L + L^2 - L^3)\, \Delta X_t]$ is the transformation that retains the unit root at the two cycles per year frequency (semiannual period);

$Z3_t = [-(1 - L^2)\, \Delta X_t]$ is the transformation retaining the unit root at the one cycle per year frequency (annual period).

If the null of the existence of two unit roots at the zero frequency is rejected, then equation [8] may be used to test for the presence of a single unit root, where the Yi_t variables have similar definitions (they are the result of using the same filters as before on X_t instead of ΔX_t). The order of the lags (value of k in the summation) is determined using diagnostic checks such that the estimated error process is approximately white noise. The test is conducted by estimating the auxiliary regressions in equations [7] and [8]. The interpretation of the results and the critical values necessary to conduct the tests can be found in Hylleberg *et al.* (1990). In our case, however, the exact critical values were obtained from the response surfaces estimated by Sansó *et al*. (1998a).

III.2. Tests with null of stationarity

Since the HEGY test takes as null the existence of a unit root at one or more seasonal frequencies, "rejection of their null hypothesis implies the strong result that the series has a stationary seasonal pattern. Due to the low power of the tests in moderate sample sizes, however, nonrejection of the null hypothesis unfortunately cannot be interpreted as evidence 'for' the presence of a seasonal unit root" (Canova and Hansen, 1995, page 237). Taking into account power considerations[8] a useful complement to the above testing procedures would be another test that takes *stationary* seasonality as the null hypothesis and the alternative to be *non-stationary* seasonality. "In this context, rejection of the null hypothesis would imply the strong result that the data are indeed non-stationary, a conclusion that the DHF or HEGY tests cannot yield. Viewed jointly with these tests, such a procedure would allow researchers a more thorough analysis of their data" (Canova and Hansen, 1995, page 238).

The starting point for these authors is a linear time series model with stationary seasonality which can be specified in two different - although mathematically equivalent - ways**:** the first one is the trigonometric representation commonly used in the time series literature;[9] the second is the dummy formulation. The former gives rise to two unit root tests at seasonal frequencies and the latter results in four tests for time variation in the coefficients of the seasonal dummy variables (quarterly data). These different tests are obtained by proper specification of the alternative hypothesis in each case. The auxiliary regression used to perform these tests is the following:

$$y_t = \mu + Z_t^{'} \beta + f_t^{'} \gamma + u_t \qquad (7)$$

[8] Power is the probability of rejecting the null hypothesis in a statistical test when it is in fact false; the power of a test of a given null clearly depends on the particular alternative hypothesis it is being tested.

[9] In this formulation a periodic sequence is represented by a Fourier series, the parameterization of the model uses Fourier coefficients, and seasonality is interpreted as a *cyclical* phenomenon (Priestley (1981), Aguirre (1995)).

where Z_t is a $(\mathrm{k} \times 1)$ vector of explanatory variables, u_t is stationary, $f_t' = [\cos(\theta_1, t), \sin(\theta_1, t), \ldots, \cos(\theta_{s/2}, \mathrm{t}), \sin(\theta s/2, \mathrm{t})]$ with $\theta_j = \frac{2\pi j}{s}$ (j =1, 2,..., s/2) and s equal to the number of yearly observations. In this way, f_t is equivalent to a set of seasonal dummy variables represented in the frequency domain.

If the alternative under consideration is 'seasonal non-stationarity' then the existence of unit roots at *all* seasonal frequencies should *simultaneously* be tested. This means that, in order to run the stability test with null of stationarity in all frequencies, the following statistic has to be calculated:

$$L_f = T^{-2} \sum_{t=1}^{T} \tilde{F}_t' (\tilde{\Omega}^f)^{-1} \tilde{F}_t$$

$$= T^{-2} \operatorname{tra}[(\tilde{\Omega}^f)^{-1} \sum_{t=1}^{T} \tilde{F}_t \tilde{F}_t']$$

where $\tilde{F}_t = \sum_{t=1}^{T} f_t \tilde{u}_t$ is a sequence of partial sums, $\tilde{u}_t$ is the set of residuals of the OLS estimation of equation (7) and

$$\tilde{\Omega}^f = \sum_{k=-m}^{m} W(k,m) \frac{1}{T} \sum_t f_{t+k} f_t' \tilde{u}_{t+k} \tilde{u}_t$$

is a consistent estimate of the variance-covariance matrix of $f_t \tilde{u}_t$ (taking into account possible heteroscedasticity and autocorrelation), where W(.,.) is a smoothing window.

If the interest is in testing for seasonal components at specific individual seasonal frequencies the relevant matrix assumes a different form and the original L statistic reduces to $L_{\theta j}$ (j = 1, 2,..., s/2) which can be computed as a by-product of the calculation of L_f. When quarterly data are used, s = 4 and two such statistics result. These are given by the quadratic form

$$L_{\theta j} = T^{-2} \sum_{t=1}^{T} \tilde{F}_{jt}' (\tilde{\Omega}_{jj}^f)^{-1} \tilde{F}_{jt}$$

where $\tilde{F}_{jt} = \sum_{t=1}^{T} f_{jt} \tilde{u}_t$, $f_{jt}' = [\cos(\theta_j, t), \sin(\theta_j, t)]$, $f_{s/2,t} = \cos(\pi, t) = (-1)^t$ and

$$\tilde{\Omega}_{jj}^f = \begin{bmatrix} \omega_{2j-1,2j-1}^f & \omega_{2j-1,2j}^f \\ \omega_{2j,2j-1}^f & \omega_{2j,2j}^f \end{bmatrix}$$

for $j < \frac{s}{2}$ and ω_{hl}^f being a characteristic element of $\tilde{\Omega}_{jj}^f$. The asymptotic distribution of the test statistics is the generalized Von Misses with degrees of freedom according to the dimension of the partial sum process.[10] "The $L_{\theta j}$ tests are useful complements to the joint test L_f. If the joint test rejects, it could be due to unit roots at any of the seasonal frequencies. The $L_{\theta j}$ tests are specifically designed to detect at which specific seasonal frequency non-stationarity emerges" (Canova and Hansen, 1995, page 242).

When testing for nonconstant seasonal patterns the more traditional model with seasonal dummy variables is used to determine if the seasonal intercepts change over time. Again, by properly choosing the form of the relevant matrix it is possible to define *s* different statistics L_a (a = 1, ..., s) which allow testing the stability of the *a*th seasonal intercept. When the objective of the test is the joint stability of the seasonal intercepts an L_J statistic is defined. However, this is a test for instability in *any* of the seasonal intercepts, in such a way that even zero-frequency movements in the series may be detected. As a result, the null hypothesis can be rejected as a consequence of the existence of long-run instability at that frequency, which is an undesirable feature of the test.[11] The modifications proposed by Canova and Hansen to cope with this problem led them back to the joint test statistic L_f defined in the first case. This result prompted the authors to remark that**:** "To put the finding in another way, we have found that either construction - testing for instability as viewed through the lens of seasonal intercepts or from the angle of seasonal unit roots - gives exactly the same joint test" (Canova and Hansen, 1995, page 243).

IV. DATA DESCRIPTION AND TEST RESULTS

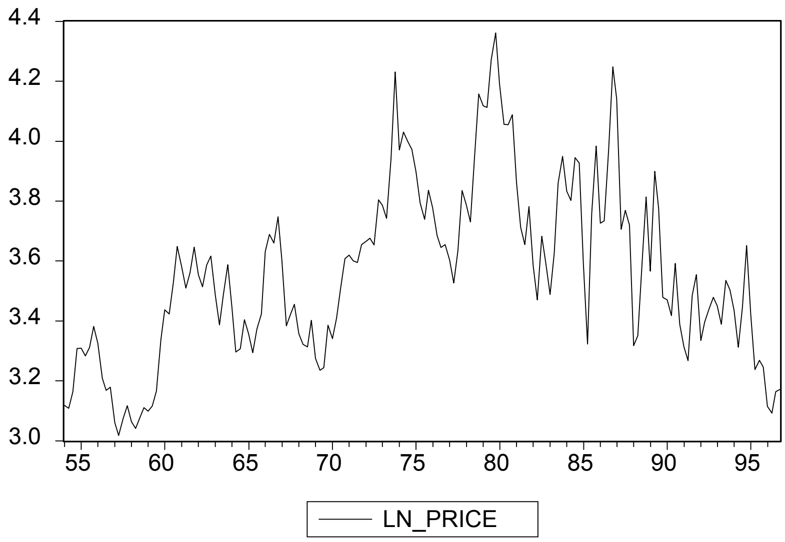
The series analyzed in this paper is the same one studied in Aguirre (1997). It is formed by quarterly prices received by producers of beef cattle in the State of São Paulo (Brazil), *per* "arroba" (15 kilograms) of live cattle, in the 1954-1996 period. The original data, published by the Agricultural Economics Institute of the Agricultural Secretariat of the State of São Paulo, are monthly average (nominal) prices. The averages represent the whole State. Those prices were deflated using the General Price Index (IGP/DI) estimated by Fundação Getúlio Vargas (FGV). The monthly real prices were averaged into quarterly prices (see Figure I).

[10] Asymptotic critical values are provided by Canova and Hansen (1995). These same authors and Hylleberg (1995) study the behavior of the test statistics, in finite samples, in the case of quarterly data; similar analysis for monthly data is presented in Sansó *et al*. (1998b).

[11] The authors recognize that this objection is also applicable to the case of the individual test statistics L_a, but the problem is far more acute with the joint test L_J.

FIGURE I

Logarithm of Quarterly Beef Cattle Prices



From Figure I it seems that the series has two trends: a positive one from the beginning up to around 1980 and a decreasing one thereafter. According to Mueller, the upward trend of real beef cattle prices in the 1954-1979 period was brought about by demand pressures. As demand increased as a result of general economic growth, supply did not follow (Mueller, 1987). The downward trend in real prices is explained by several factors. On the supply side, there were important improvements in production technologies, the most important of which seems to have been the adoption of new varieties of pastures. On the demand side, there was a significant loss of purchasing power of consumers due to the economic recession of the 1981-1984 period. That loss was more significant for higher salaries,[12] affecting the high income-elasticity group of consumers with highest rates of consumption of beef in the country. The seasonal fluctuations observed in the price series of the State of São Paulo are due to the alternate occurrence of rain and drought seasons which affect the availability of grass and the supply of cattle (Margarido *et al.*, 1996).[13] This characteristic is similar to that observed in the beef market of the U.S. where cyclical annual variations in the real prices are also attributed to supply fluctuations.

If a linear trend is fit to the data with a model like $\log P_t = \alpha + \beta t + \varepsilon_t$, it is possible to use the Chow test (Chow, 1960) to check whether the trend coefficient may be regarded as constant over the whole period. This test produces significant F statistics - indicating an structural break - for several dates of the breakpoint. However, the maximum F-value occurs when the series is split at the third quarter of 1979. This partitioning of the data will be used in this section to run seasonal unit root tests for each data subset, together with the whole series, in order to check for the stability of the results.

[12] The rules dictated by the federal government to index public servant salaries to inflation implied real loses which were directly proportional to salary levels. During the recession years the private sector applied the same rules (Aguirre, 1984).

[13] The price series of fat cows, also sold by weight, presents these same intra-annual movements. The price series of other types of animals (calves, yearlings, 'unfinished' steers, etc.) sold on a 'per head' basis, do not show any seasonal variation.

In Table II we present the relevant coefficients to perform the several tests included in Table I. Since our series presents trends, all tests of the null hypothesis of the existence of *a single unit root* at the zero frequency were performed with two different model specifications**:** (a) not including and (b) including a deterministic trend.[14] Both procedures give similar results in all the different tests and, for this reason, no detailed information about this point is presented. The tests were run on the X_t series (the logarithm of the prices) and on some transformations obtained applying specific filters to it. In the first two equations the ADF test is run on the first difference of X_t and on X_t itself, respectively. Equation [1] tests for the presence of two unit roots at the zero frequency while *it is assumed* that there are no seasonal unit roots in the series. After rejecting the existence of two unit roots at the zero frequency (1% level of significance) the same type test checks for the presence of a single unit root. This time the null is not rejected (both with and without a deterministic trend) not even at the 10% level. It is worth noting that in both cases above, the absence of seasonal unit roots is stated in the null as well as in the alternative hypothesis, since the ADF test is not designed to check for the existence of these roots. The conclusion from these tests is that the series is SI(1,0).

TABLE II

Results of the tests with null of nonstationarity

Test	Remarks	Test statistic
[1] ADF for ΔX_t series [a]		-5.95^{***}
[2] ADF for X_t series [a] ADF for X_t series [b]		–1.99 –1.66
[3] ADF for $\Delta_4 X_t$ series [a]		-4.46^{***}
[4] ADF for $S(L)X_t$ series [a] ADF for $S(L)X_t$ series [b]		–1.72 –0.35
[5] DHF for ΔX_t series [a]		-11.67^{***}
[6] DHF for X_t series [a] DHF for X_t series [b]		–2.98 –2.74
[7] HEGY for ΔX_t series [a]	π_1 π_2 π_3 π_4 $\pi_3 \cap \pi_4$	-6.88^{***} -8.05^{***} -5.08^{***} 0.77 13.55^{***}
[8] HEGY for X_t series [a]	π_1 π_2 π_3 π_4 $\pi_3 \cap \pi_4$	–1.99 -9.89^{***} -4.24^{***} -3.48^{***} 14.37^{***}
HEGY for X_t series [b]	π_1 π_2 π_3 π_4 $\pi_3 \cap \pi_4$	–1.66 -9.87^{***} -4.23^{***} -3.44^{***} 14.22^{***}

[a] With a constant and seasonal dummies.
[B] With constant, dummies and a deterministic trend.
[***] Significant at 1% level.

[14] While the unnecessary inclusion of this term only slightly reduces the power of the test, its omission may bring about more serious problems to the testing procedure.

The same test structure can be applied to the seasonally differenced series (equation[3]). In that case, the ADF test checks for the presence of two unit roots at the zero frequency while *admitting the existence of seasonal unit roots*. After rejecting the existence of two roots, at the 1% level of confidence, the presence of a single one can be checked by running the same test on the seasonal sum of X_t. In this case we cannot reject the null hypothesis at the usual confidence levels and we conclude that the series is SI(1,1).

All these results imply the presence of a single unit root at the zero frequency. The possible existence of seasonal unit roots at some (or all) of the seasonal frequencies was not put under check because the ADF test is not designed to do that. The DHF test, on the contrary, checks for both kinds of roots. If this test is run on the first differences of the series (see equation [5]), the null hypothesis SI(2,1) is tested against the SI(1,0) alternative. The significant result shown in Table III rejects this null at the 1% level. To check if the series is completely stationary the DHF test is run with the series in levels - as indicated by equation [6] - testing the null hypothesis SI(1,1) against SI(0,0). According to our results the null is not rejected. Given the results produced by the DHF test it seems clear that the series under scrutiny presents a unit root at the zero frequency, but the evidence about seasonal unit roots is not clear-cut due to the inherent deficiencies of this type of test commented on in the last section.

Turning to the HEGY test, Table I shows the structure of the auxiliary regressions which may be estimated with the ΔX_t and X_t series.[15] Different null hypotheses can be tested with each auxiliary regression depending on the *a priori* assumptions made about the coefficients π_i. At the same time, for a given null different alternative hypotheses result depending on what coefficients are equal to (or different from) zero and which ones are subject to test. Ilmakunnas (1990) presents other combinations of hypotheses not included in Table I. The results obtained from the estimation of equation [7] point to the rejection of the null SI(2,1) in favor of SI(1,0). The other alternative cannot be chosen since the $\pi_2 = \pi_3 = \pi_4 = 0$ condition is not fulfilled in this case. In the next step, the estimation results of equation [8] allow us to test SI(1,1) against SI(1,0) by testing the coefficients π_i (i = 2, 3, 4), since π_1 is not significantly different from zero. In that case we reject the null and conclude that the series is SI(1,0).

Concerning the results obtained in the application of the CH methodology (see Table III), we run the tests on the first differences of the X_t series (the logarithms of the prices), and do not include a lagged dependent variable among the regressors. The results show that the series displays a statistically significant seasonal pattern which is not constant. Table III reports the values of the L_π and $L_{\pi/2}$ statistics for stability tests at seasonal frequencies, and the joint test statistic L_f for two different lag windows (m = 5 and m = 9). The significant value of L_π means that the null hypothesis of stationarity at that frequency is rejected, which implies the presence of a unit root at frequency π (semiannual period). Using the third definition of a seasonal integrated process mentioned in Section II, this result means that the series is $I_\pi(1)$. As a consequence, the statistic for the joint test is also significant indicating that the seasonal pattern has changed over the sample. This result is in contradiction with that obtained before with the HEGY methodology [equation (8) of Table II] which rejects the null hypothesis stating the existence of a seasonal unit root at frequency π. We will comment on this point in the concluding section.

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[15] The definitions of the *Zi* and *Yi* variables are given in Section III.1.

TABLE III

Results of the tests with null of stationarity

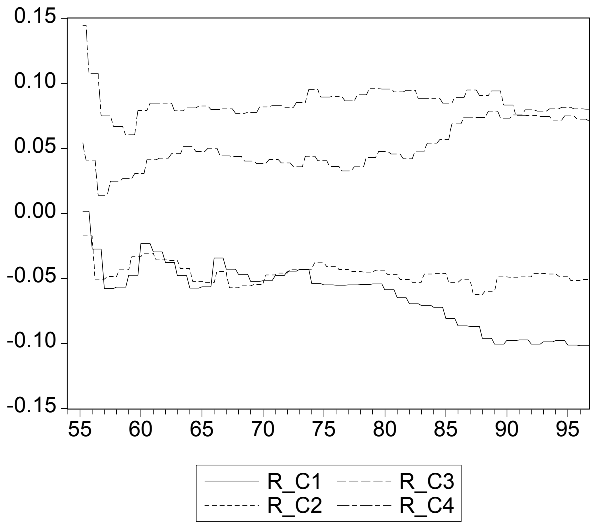
	Critical values	Test statistics	
		m = 5	m = 9
Two cycles per year (frequency π)	0.75	$L_{\pi} = 1.54^{**}$	$L_{\pi} = 1.30^{**}$
One cycle per year (frequency π/2)	0.47	$L_{\pi/2} = 0.08$	$L_{\pi/2} = 0.09$
Joint test for both seasonal frequencies	1.01	$L_f = 1.77^{**}$	$L_f = 1.56^{**}$
Quarter 1	0.47	$L_1 = 0.66^{**}$	$L_1 = 0.62^{**}$
Quarter 2	0.47	$L_2 = 0.26$	$L_2 = 0.27$
Quarter 3	0.47	$L_3 = 0.89^{**}$	$L_3 = 0.85^{**}$
Quarter 4	0.47	$L_4 = 0.03$	$L_4 = 0.03$

** Significant at the 5% level.

Looking at the values of the L_i (i = 1,...,4) statistics to test the stability of each separate dummy coefficient we see that L_1 and L_3 are significant, meaning that changes occurred in quarters 1 and 3. It is interesting to compare these results about individual dummy stability tests with those we obtain from recursive estimates of the quarterly dummy coefficients in the $(1-L)X_t = \sum_{i=1}^{4} \alpha_i Di_t + \varepsilon_t$ model (see Figure II). Despite the reduced scale, the graph shows that the first coefficient decreases while the third one increases during this period (both movements are stronger during the 1980's), a result that coincides with the tests that reject the constancy of the coefficients for the first and third quarter dummies.

FIGURE II

Recursive Estimates of Seasonal Dummy Coefficients



Finally, we report that after dividing the series in two parts as discussed in the last section, all the tests here presented were performed for both subperiods and that the results we obtained were qualitatively the same as those corresponding to the complete series.

V. CONCLUSIONS

In this paper we use different techniques to evaluate the seasonal characteristics of the quarterly series of beef cattle prices in the State of São Paulo. Our results are compared with those obtained by Aguirre (1997) who studied the same series. In both papers the main objective is to determine if the seasonal variation present in the data is deterministic or stochastic and, in the second case, if it is stationary or if the process contains seasonal unit roots. However, the testing strategy followed in this paper presents some differences which were already reported and which will be summarized in this last section.

The Dickey-Pantula approach was followed in all the tests performed in order to check for the presence of two unit roots at the zero frequency. This procedure is different from that followed by Aguirre (1997) who used the testing sequence recommended by Charemza and Deadman (1992). Another difference is that we obtain definite results from the use of the DHF test while Aguirre (1997) did not. This may be explained by the different model specifications used in each case, since we include seasonal dummy variables.

Concerning the application of the HEGY tests we checked for the possible presence of two unit roots at the zero frequency and rejected this hypothesis. However, the hypothesis of the existence of a single unit root in that frequency cannot be rejected. In relation to the possible existence of seasonal unit roots we conclude that the seasonality of the series is partly deterministic and partly stationary stochastic. These last two results coincide with those reported by Aguirre (1997).

Another novelty in our testing procedure is the use of the CH tests which reveal that the series displays a statistically significant seasonal pattern with changing coefficients of the seasonal dummy variables corresponding to the first and third quarters. This last result is confirmed by the estimation of recursive regressions.

One result of the CH tests that does not agree with those of the HEGY-type tests is the existence of a seasonal unit root at frequency π (semiannual period). These two contradictory conclusions are rather puzzling since both of them are the result of rejecting the corresponding null hypothesis of each test. As a consequence, neither result can be attributed to lack of power of any of the tests. Actually, in spite of Canova and Hansen's contention that their tests are more powerful, some results obtained in the last years show that more than to increase the power of the test - a result which shows mixed evidence - the CH methodology is a complementary technique that allows the researcher to look at the problem from a different perspective. With the new null we only reject the hypothesis of stationarity if the available evidence against it is very strong. So, by using both tests neither null is privileged over the other. If we obtain similar results with both types of tests then our conclusion is that there is strong evidence in favor of the corresponding hypothesis. If the results are contradictory (like in our case for the unit root of frequency π) either the data do not contain enough information to discriminate between these hypotheses or the DGP may be of a different nature from the models we are using - for example, they may be nonlinear.[16] One possible way to look into this problem would be to conduct a Monte Carlo type experiment. That would involve simulating the functioning of the process a 'reasonably' large number of times using a nonlinear model[17] and utilizing this data to establish the empirical frequency with which each type of test rejects the corresponding null hypotheses. Such a task, however, goes beyond the objectives of this paper.

For all these reasons it is worth to conclude with the following quotation: "In view of these results, we agree with the advise of Canova and Hansen (1995) and Hylleberg (1995) in the sense that it is very convenient to simultaneously use the tests with null of seasonal nonstationarity together with the CH tests. If there is agreement in the evidence obtained from both types of tests, then this can be interpreted as strong evidence. On the contrary, if those methodologies produce different results, then detailed analyses are needed because it is evident that the data do not allow to properly discriminate between the trend-stationary hypothesis and the difference-stationary case" (Sansó *et al.,* 1998b).

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[16] See Aguirre and Aguirre (forthcoming).

[17] For example, one of the models presented by Aguirre and Aguirre (forthcoming) could be used for this objective.

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